Spinor-Vector duality in smooth heterotic compactifications

Martín Hurtado Heredia
Joint work w/ Stefan Groot Nibbelink Alon Faraggi
based in 2111.10407, 2103.13442 , 2103.14684

String Pheno 2022

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- Initial observation: $\exists$ symmetry underlying the distribution of vacua of free fermionic realization of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds [Carlo Angelantonj, Alon E. Faraggi, Mirian Tsulaia 1003.5801] [Alon E. Faraggi, Costas Kounnas John Rizos 0712.0747]

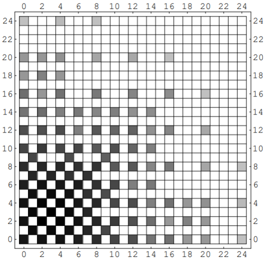


Figure: Density plot for free fermionic $SO(10)$ models. (similar for SO(12) models)

Spinor-vector duality

Dual models are related by $\#\boldsymbol{V} \leftrightarrow \#(\mathbf{S} + \bar{\mathbf{S}})$ of the underlying $SO(2N)$ GUT symmetry group.

Origin of SVD

- FF → exchange in generalized GSO phases
- Orbifold language → Change in **Discrete torsion** of the $Z_2 \times Z_2$ partition function

Importance/Implications of SVD

Same logic than Mirror Symmetry:

- In [Cumrun Vafa, Edward Witten, 9409188] mirror symmetry in a $T^6/Z_2 \times Z_2$ → change in ε of the orbifold twists → change in the internal space moduli $h^{1,1} \leftrightarrow h^{2,1}$
- In the heterotic string we have moduli from the gauge degrees of freedom, (i.e. W_i) and the SVD arises as a change in ε between the orbifold twist and the W_i

Relation to T-duality

- Generates mirror symmetry for orbifolds [paper]
- Self-dual point enhancement

Question

How this SVD manifest itself in the effective limit?

- Understand discrete torsion in smooth compactifications...

Constraint on Spinor-Vector Dualities in Six Dimensions

When studying the 6D case we came to a fundamental constraint (no matter of the orbifold/smooth regime)

Main result

Any six dimensional $\mathcal{N} = 1$ supersymmetric effective field theory with the numbers of vectors N_V and of spinors N_S (of either chirality) of some $SO(2N)$ gauge group are constrained by an anomaly condition to

$$N_V = 2^{N-5} N_S + 2N - 8, \quad \text{for } N \geq 3$$

This forces the only posible SVduals models to be **self-dual** in $6D$

To derive we use that the anomaly polynomial I_8 should vanish. For charged fermions it takes the form:

$$I_{8|R} = \widehat{A}\left(R_2\right) \mathrm{ch}_R\left(F_2\right)\Big|_8$$

with

$$\mathrm{ch}_R\left(F_2\right) = \mathrm{tr}_R\left[e^{i\frac{F_2}{2\pi}}\right]$$

There are three different contributions whose total sum should vanish

- N_V Hyper multiplets in the vector representation:

$$I_{8|V} \supset N_V \frac{1}{4!} \operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^4$$

- Gauge multiplet in the adjoint representation:

$$I_{8|Ad} \supset -\frac{1}{4!} \left[(2N-8) \operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^4 + 3 \left(\operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^2 \right)^2 \right] \supset -(2N-8) \frac{1}{4!} \operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^4$$

- N_S Hyper multiplets in the (conjugage) spinor representation:

$$I_{8|S} \supset N_S \frac{1}{4!} 2^{N-5} \left[-\operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^4 + \frac{3}{4} \left(\operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^2 \right)^2 \right] \supset -2^{N-5} N_S \frac{1}{4!} \operatorname{tr}_V \left(i \frac{F_2}{2\pi} \right)^4$$

This result was confirmed by

- Revision of the spectra of $6D$ orbifold in the literature
- Construction of $6D$ free fermion models (only SVD in the E_7 self-dual point)
- Computation of the spectra of generic $K3$ surfaces with arbitrary line bundle backgrounds

- We start with orbifold models of $T^4/\mathbb{Z}_2 \times S^1$ with a Wilson line on the additional circle.
- We then consider the resolution of this orbifold to a smooth $\mathrm{K3} \times S^1$ realisation and investigate how this effects the spinor-vector duality

Main problem

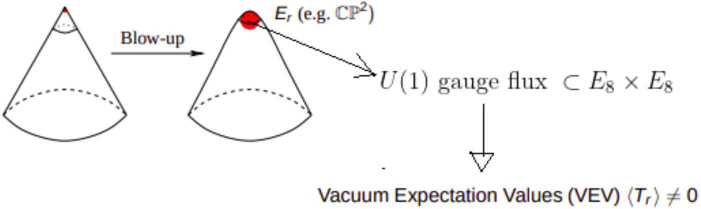
- In the orbifold → SVD comes from switching on/off ε between the orbifold twist and W
- Changing ε changes the projection condition of the Wilson line: $W \cdot P \equiv 0. \Leftrightarrow W \cdot P_{sh} \equiv \frac{1}{2}k$:
- Thus, untwisted states remains the same but twisted states projection condition is reversed
- In this case the gauge group remains the same
- In general this modification does not make sense for smooth compactifications (twisted and untwisted states?)

(Provisional) Solution

Iff resolution is unique we can track the twisted and untwisted states of the resolution model back to the orbifold → **Use the same modification of the projection condition that we use for orbifolds**

There are some subtleties in this solution

- Moving from the orbifold point to smooth resolutions blowup modes have to be selected.
- These are twisted states which develop VEVs inducing the blowup of the orbifold singularities.
- Since the Wilson lines on (additional) cycles lead to projections of the twisted spectrum with or without torsion, the selected blowup modes may not be in the spectrum anymore, which leads to various complications.
- A prime one being that the choice of the torsion phase affects which blowup modes are available.



Torsion Phase (ϵ)	Without ($\epsilon = 0$)	With ($\epsilon = 1$)
Orbifold Gauge Group Spectrum	$SU(2)_1 \times SU(2)_2 \times SO(12) \times SO(16)'$ $(2,2,12)(1) + 16\,\frac{1}{2}(1,2,12)(1)$ $+32\,(2,1,1)(1) + 4(1,1,1)(1)$	$SU(2)_1 \times SU(2)_2 \times SO(12) \times SO(16)'$ $(2,2,12)(1) + 16\,\frac{1}{2}(1,1,32)(1)$ $+4(1,1,1)(1)$
Blowup Modes $P_{\text{sh},\alpha} = V_\alpha$ Gauge Group Spectrum	$(\frac{1}{2}, -\frac{1}{2}, 1, 0^5)(0^8)$ $SU(2) \times SO(10) \times SO(16)'$ $2\,(2,10)(1) + 36\,(2,1)(1)$ $+14\,(1,10)(1) + 14\,(1,1)(1)$	$(0^2, \frac{1}{2}^6)(0^8)$ $SU(2)_1 \times SU(2)_2 \times SU(6) \times SO(16)'$ $2\,(2,2,6)(1) + 14\,(1,1,15)(1)$

Figure: This table summarises how a spinor-vector duality is visible in orbifold and resolution models. Since the resolutions depend on the choice of blowup modes, their gauge groups and therefore their spectra make this duality less apparent.

When studying the 4D case we arrive to an unsettling scenario:

Problem

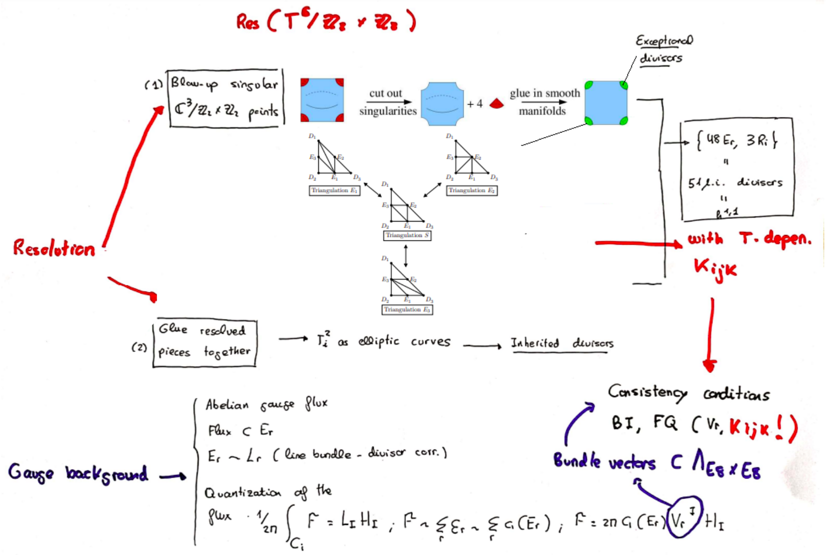
Strong triangulation dependence: 64 $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ singular points, with four different ways to resolve each one $\longrightarrow \frac{4^{64}}{3!4!^3} \approx 4.10\cdot 10^{33}$ different resolutions!

- Otherwise you are forced to choose a T and study a particular model from the beginning...

To study the resolved model in general we would like to obtain triangulation-independent expressions for the consistency conditions and phenomenological quantities

This can be seen as the second part of [Michael Blaszczyk, Stefan Groot Nibbelink, Fabian Ruehle, Michele Trapletti, Patrick K.S. Vaudrevange 1007.0203] in which a MSSM-like model was built in a particular triangulation

4D: A detour into Res$\left(\frac{T^6}{\mathbb{Z}_2\times\mathbb{Z}_2}\right)$



Solution: Parameterizing Triangulations

The idea is to absorb the dependence on the triangulation in some functions s.t. we obtain expressions independent of any triangulation

- Define the following functions: $\delta^T_{\alpha\beta\gamma} = \begin{cases} 1 & \text{if triangulation } T \text{ is used,} \\ 0 & \text{if other triangulation is used,} \end{cases}$
  of (α,β,γ) for the four possible triangulations dubbed $T = S$, E_1, E_2 and E_3.
- Define also the following Δ functions
$$\begin{aligned} \Delta^1_{\alpha\beta\gamma} &= -\delta^{E_1}_{\alpha\beta\gamma} + \delta^{E_2}_{\alpha\beta\gamma} + \delta^{E_3}_{\alpha\beta\gamma} \,, \\ \Delta^2_{\alpha\beta\gamma} &= \delta^{E_1}_{\alpha\beta\gamma} - \delta^{E_2}_{\alpha\beta\gamma} + \delta^{E_3}_{\alpha\beta\gamma} \,, \\ \Delta^3_{\alpha\beta\gamma} &= \delta^{E_1}_{\alpha\beta\gamma} + \delta^{E_2}_{\alpha\beta\gamma} - \delta^{E_3}_{\alpha\beta\gamma} \,. \end{aligned}$$

It follows immediately that
$1 - \Delta^1_{\alpha\beta\gamma} - \Delta^2_{\alpha\beta\gamma} - \Delta^3_{\alpha\beta\gamma} = \delta^S_{\alpha\beta\gamma}$, $\quad 1 - \Delta^i_{\alpha\beta\gamma} = 2\delta^{E_i}_{\alpha\beta\gamma} + \delta^S_{\alpha\beta\gamma}$ and
$\Delta^2_{\alpha\beta\gamma} + \Delta^3_{\alpha\beta\gamma} = 2\delta^{E_1}_{\alpha\beta\gamma}$, $\quad \Delta^1_{\alpha\beta\gamma} + \Delta^3_{\alpha\beta\gamma} = 2\delta^{E_2}_{\alpha\beta\gamma}$, $\quad \Delta^1_{\alpha\beta\gamma} + \Delta^2_{\alpha\beta\gamma} = 2\delta^{E_3}_{\alpha\beta\gamma}$

Res$\left(\frac{T^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}\right)$

Now we plug this functions into our intersection numbers and its integrated second Chern classes and that allow us to obtain triangulation independent consistency conditions

Topological data

$$R_1 E^2_{1,\beta\gamma} = R_2 E^2_{2,\alpha\gamma} = R_3 E^2_{3,\alpha\beta} = -2 \,, \qquad R_1 R_2 R_3 = 2 \,,$$

$$E_{1,\beta\gamma} E^2_{2,\alpha\gamma} = E_{1,\beta\gamma} E^2_{3,\beta\gamma} = -1 + \Delta^1_{\alpha\beta\gamma} \,, \qquad E^3_{1,\beta\gamma} = \sum_\alpha \left(1 + \Delta^1_{\alpha\beta\gamma}\right) ,$$

$$E_{2,\alpha\gamma} E^2_{1,\beta\gamma} = E_{2,\alpha\gamma} E^2_{3,\beta\gamma} = -1 + \Delta^2_{\alpha\beta\gamma} \,, \qquad E^3_{2,\alpha\gamma} = \sum_\beta \left(1 + \Delta^2_{\alpha\beta\gamma}\right) ,$$

$$E_{3,\alpha\beta} E^2_{1,\beta\gamma} = E_{3,\alpha\beta} E^2_{2,\alpha\gamma} = -1 + \Delta^3_{\alpha\beta\gamma} \,, \qquad E^3_{3,\alpha\beta} = \sum_\gamma \left(1 + \Delta^3_{\alpha\beta\gamma}\right) ,$$

$$E_{1,\beta\gamma} E_{2,\alpha\gamma} E_{3,\beta\gamma} = 1 - \Delta^1_{\alpha\beta\gamma} - \Delta^2_{\alpha\beta\gamma} - \Delta^3_{\alpha\beta\gamma} \,.$$

$$\Downarrow$$

$$c_2 R_1 = c_2 R_2 = c_2 R_3 = 24 \,, \quad c_2 E_{1,\beta\gamma} = \sum_\alpha \left(1 - 2\Delta^1_{\alpha\beta\gamma}\right) ,$$

$$c_2 E_{2,\alpha\gamma} = \sum_\beta \left(1 - 2\Delta^2_{\alpha\beta\gamma}\right) , \quad c_2 E_{3,\alpha\beta} = \sum_\gamma \left(1 - 2\Delta^3_{\alpha\beta\gamma}\right) .$$

Consistency conditions

$$\mathcal{V}_{1,\beta\gamma} \cdot \mathcal{V}_{2,\alpha\gamma} = \mathcal{V}_{1,\beta\gamma} \cdot \mathcal{V}_{3,\alpha\beta} = \mathcal{V}_{2,\alpha\gamma} \cdot \mathcal{V}_{3,\alpha\beta} = \tfrac{1}{2}\left(\mathcal{V}^2_{1,\beta\gamma} + \mathcal{V}^2_{2,\alpha\gamma} + \mathcal{V}^2_{3,\alpha\beta}\right) - 2 \,.$$

$$\sum_\beta \mathcal{V}^2_{1,\beta\gamma} = \sum_\gamma \mathcal{V}^2_{1,\beta\gamma} = 6 \,, \quad \sum_\alpha \mathcal{V}^2_{2,\alpha\gamma} = \sum_\gamma \mathcal{V}^2_{2,\alpha\gamma} = 6 \,, \quad \sum_\alpha \mathcal{V}^2_{3,\alpha\beta} = \sum_\beta \mathcal{V}^2_{3,\alpha\beta} = 6 \,.$$

$$\mathcal{V}^2_{a,\mu\nu} = \tfrac{3}{2} \qquad \Rightarrow \qquad \mathcal{V}_{a,\mu\nu} \cdot \mathcal{V}_{b,\rho\sigma} = \tfrac{1}{4} \,;$$

$$2\,\mathcal{V}_{i,\mu\nu} \cong 0 \,, \quad \sum_\rho \mathcal{V}_{i,\rho\nu} \cong 0 \,, \quad \sum_\rho \mathcal{V}_{i,\mu\rho} \cong 0 \,, \quad \mathcal{V}_{1,\beta\gamma} + \mathcal{V}_{2,\alpha\gamma} + \mathcal{V}_{3,\alpha\beta} \cong 0$$

$\mathrm{Res}(\frac{T^6}{\mathbb{Z}_2\times\mathbb{Z}_2})$

- We also obtain relatively compact expressions for the volumes of curves, divisors and the manifold as a whole
- Non-trivial check! Third Chern class equals 96 independently of any triangulation choice

$$c_3 = -\frac{1}{8}\sum_{\alpha,\beta,\gamma}\left\{E_{1,\beta\gamma}\left(E^2_{2,\alpha\gamma}+E^2_{3,\alpha\beta}\right)+E_{2,\alpha\gamma}\left(E^2_{1,\beta\gamma}+E^2_{3,\alpha\beta}\right)+E_{3,\alpha\beta}\left(E^2_{1,\beta\gamma}+E^2_{2,\alpha\gamma}\right)\right\}$$

$$+\frac{1}{4}\sum_{\beta,\gamma}E^3_{1,\beta\gamma}+\frac{1}{4}\sum_{\alpha,\gamma}E^3_{2,\alpha\gamma}+\frac{1}{4}\sum_{\alpha,\beta}E^3_{3,\alpha\beta}.$$

$$c_3 = \frac{1}{4}\sum_{i,\alpha,\beta,\gamma}\left(1+\Delta^i_{\alpha\beta\gamma}\right)-\frac{1}{4}\sum_{i,\alpha,\beta,\gamma}\left(-1+\Delta^i_{\alpha\beta\gamma}\right)=96$$

Res($\frac{T^6}{\mathbb{Z}_2\times\mathbb{Z}_2}$): Models

Results and applications

- If the flux quantisation conditions are satisfied for a given specific choice of triangulation, they are, in fact, fulfilled for any configuration of triangulations.
- Computation the spectra different line bundle models in all triangulations explicitly, it was confirmed that the full chiral spectra are always integral.
- Analysis of the spectrum in more complicated models → a variant of the Blasczczyks GUT model with four Wilson lines of which three were set equal.
  - The full spectrum computed in the S triangulation everywhere is integral and free of nonAbelian anomalies.
  - But also all the local difference multiplicities measuring the jumps in the local spectra at specific resolved singularities are always integral and free of nonAbelian anomalies (as the jumping spectra were all vectorlike in this particular example).
- Study SVD in this manifold without triangulation dependence
- Application to other toroidal orbifolds

Future directions: 4D: GLSMs of $Res((\frac{T^6}{\mathbb{Z}_2\times\mathbb{Z}_2}))$ with torsion

- To study discrete torsion in we can use gauged linear sigma models (GLSMs) which allows to study both orbifold and smooth regime in the same formalism
- Build GLSMs for the torsion case of the non-compact $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ resolutions. $\rightarrow$ Difference between a torsion and non-torsion model obtained by doing a chiral superfield dependent redefinition of the fermi superfields. $\rightarrow$ Field dependent FI terms identified. In terms of homogeneous coordinates this implies the system has torsion.
- Currently compact case, vector bundle...